Chapter 10 Permutation and Combination

1. (a) (i)Find how many different 4-digit numbers can be formed using the digits 2, 3, 5, 7, 8 and 9, if each digit may be used only once in any number.

$6P_4 = 360$

[1]

(ii) How many of the numbers found in **part (i)** are divisible by 5?

$\_\_\_\underline{5}$ $5P_3 = 60$

[1]

(iii) How many of the numbers found in **part (i)** are odd and greater than 7000?

$\underline{7}\_\_\underline{3,5,9}$ $4P_2 \times 10 = 120$

$\underline{8}\_\_\underline{3,5,7,9}$

$\underline{9}\_\_\underline{3,5,7}$

[4]

(b) The number of combinations of n items taken 3 at a time is $92n$. Find the value of the constant n.

$${}^nC_3 = 92n$$

$$\frac{n!}{(n-3)! \times 3!} = 92n$$

$$\frac{n(n-1)(n-2)(n-3)\times\cdots\times 1}{(n-3)\times\cdots\times 1 \times 3\times 2\times 1} = 92n$$

$$\frac{n^2-3n+2}{6} = 92$$

$$n^2-3n+2 = 552$$

$$n^2-3n-550 = 0$$

$$(n-25)(n+22) = 0$$

$n = 25$ or $n = -22$ (Reject)

[4]

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2. (a)(i) Find how many different 5-digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number.

$^6P_5 = 720$

[1]

(ii) How many of the numbers found in **part (i)** are not divisible by 5?

_ _ _ _ 5

$^5P_4 = 120$

$720 - 120$

$= 600$

[1]

(iii) How many of the numbers found in **part (i)** are even and greater than 30 000?

3 _ _ _ 2,8 $\quad ^4P_3 \times 7 = 168$

5 _ _ _ 2,8

7 _ _ _ 2,8

8 _ _ _ 2

[4]

(b) The number of combinations of n items taken 3 at a time is 6 times the number of combinations of n items taken 2 at a time. Find the value of the constant n.

$^nC_3 = 6\ ^nC_2$

$\frac{n!}{(n-3)! \times 3!} = \frac{3}{} C \times \frac{n!}{(n-2)! \times 2!}\ 1$

$\frac{1}{(n-3)(n-4) \times \ldots \times 1 \times 6} = \frac{3}{(n-2)(n-3) \times \ldots \times 1}$

$n - 2 = 18$

$n = 20$

[4]

3. (a) In an examination, candidates must select 2 questions from the 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done.

$^5C_2 \times {}^8C_4 = 700$ [2]

(b) The digits of the number 6 378 129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done.

[2]

_ _ _ _ _ _ $\underline{6, 8, 2}$

$6! \times 3 = 2160$

4. (a)(i) Find how many different 5-digit numbers can be formed using five of the eight digits 1, 2, 3, 4, 5, 6, 7, 8 if each digit can be used once only.

$^8P_5 = 6720$ [2]

(ii) Find how many of these 5-digit numbers are greater than 60 000.

[2]

$\underline{6}\_\_\_\_ \quad {}^{7}P_4 \times 3 = 2520$

7

8

(b) A team of 3 people is to be selected from 4 men and 5 women. Find the number of different teams that could be selected which include at least 2 women.

[2]

M	W
1	2
0	3

$${}^{4}C_1 \times {}^{5}C_2 + {}^{5}C_3$$

$$= 40 + 10$$

$$= 50$$

5. (a) (i) Find how many different 4-digit numbers can be formed using the digits 1, 3, 4, 6, 7 and 9. Each digit may be used once only in any 4-digit number.

$^6P_4 = 360$ [1]

(ii) How many of these 4-digit numbers are even and greater than 6000?

6 _ _ 4 $\quad {}^4P_2 \times 5 = 60$

7 _ _ 6,4

9 _ _ 6,4 [3]

(b) A committee of 5 people is to be formed from 6 doctors, 4 dentists and 3 nurses. Find the number of different committees that could be formed if
(i) there are no restrictions,

$^{13}C_5 = 1287$ [1]

(ii) the committee contains at least one doctor,

total - no doctor

$1287 - {}^7C_5 = 1266$ [2]

(iii) the committee contains all the nurses.

$^3C_3 \times {}^{10}C_2 = 45$ [1]

6. (a) Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively.

$^{12}C_3 \times {}^{9}C_4 \times {}^{5}C_5$

$= 27720$

[3]

(b) 4-digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if

(i) there are no restrictions,

$^{5}P_4 = 120$

[1]

(ii) the number is even,

$\_\ \_\ \_\ \underline{2,8}$

$= {}^{4}P_3 \times 2 = 48$

[1]

(iii) the number is greater than 7000 and odd.

$\underline{7}\ \_\ \_\ \underline{3,9}$ $\quad {}^{3}P_2 \times 7 = 42$

$\underline{8}\ \_\ \_\ \underline{3,7,9}$

$\underline{9}\ \_\ \_\ \underline{3,7}$

[3]

7. A 4-digit code is to be formed using 4 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find how many different codes can be formed if

(a) there are no restriction

$^9P_4 = 3024$

[1]

(b) only prime numbers are used,

$4! = 24$

[1]

(c) two even numbers are followed by two odd numbers,

E E O O

$^4P_2 \times {}^5P_2 = 240$

[2]

(d) the code forms an even number.

_ _ _ 2, 4, 6, 8

$^8P_3 \times 4 = 1344$

[2]